ETHICS IN MATHEMATICS
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University of Cambridge

Four Levels of Ethical Engagement

We observe that the types of reactions of mathematicians to ethical issues proper to mathematics differ, and we attempt a sensible categorisation of the states of social and ethical responsibility of mathematicians.

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Abstract

In this *discussion document* we take as given that there exist ethical issues in mathematics, and that all mathematicians need to be aware of their particular professional social and ethical responsibilities. This view is not widely shared amongst professional mathematicians, however. We observe that ethical awareness and responses amongst mathematicians are complicated and not binary on-off: both the sensitivity to issues and the kinds of responses they engender are highly variable. We categorise these with four different levels or kinds of ethical engagement, and attempt a description of the characteristics of each; this is a prolegomena to a discussion as to *why* mathematicians have the social and ethical behaviour that we observe.

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Ethics in Mathematics: Four Levels of Ethical Engagement[1]

Introduction

In other discussion papers in this series, and elsewhere as part of our work on the Cambridge University Ethics in Mathematics Project[2], we address the many different ways in which mathematicians – in industry and academe, research and practice – may find themselves facing clear, or alternatively very subtle, ethical issues. Indeed, several of these ethical issues are inherent to mathematics. For what follows, we would like to presume that the reader at least agrees that mathematics and mathematicians face mathematics-specific ethical issues – just as lawyers, medics, biologist, chemists, physicists and engineers all may face subject specific ethical issues as part of the study and practice of these disciplines. Nobody[3] questions that doctors and lawyers need to have some ethical training, guidelines, and a sensitivity to difficult issues their practice may encounter. We assert the *premise* here that the same is true for mathematicians. There will be other discussion papers in this series that examine this premise.

This premise is far from universally accepted by mathematicians, in our experience. Many mathematicians do not feel that there is ethics in mathematics, nor that there is any need for subject-specific training in ethics for mathematicians.

As we have developed our analysis of many different ethical problems mathematicians may face, and the different places in social space where mathematics needs to consider issues of social responsibility and ethical consequences, we have noticed different kinds of reactions or instantiations of ethical awareness or ethical consciousness amongst mathematicians. These we describe as different 'levels' of ethical engagement or ethical activity, going from 'level 0' (which is *no* ethical engagement, a denial of there being ethics in mathematics) to 'level 4' which is an altogether difficult and burdensome professional social responsibility. We present these in this paper.

We wish to show that these *levels* (or stages, or states) of ethical consciousness are just a categorisation that – momentarily – we find instrumentally useful in organising our thinking and our discussion about ethics, mathematics, and mathematicians. We do not want to get into a detailed argument about shades of ethical levels (i.e., defining level 2.8/b§5…); that is not our point. And we do not want to suggest that there is a 'ladder', whereby mathematicians start at level 0 and completing their ascent to ethical nirvana at level 4. We presume that in time we will find further states or kinds of ethical involvement amongst mathematicians – this is a categorisation we hope will become broader or deeper with time. What we really want to do is to

[1] The authors wish to acknowledge extensive discussions with Dennis Müller, and the careful editing and detailed commentary of Toby Clifton. Despite all attempts by others to correct our mistakes, misunderstandings, and bad English the authors are sadly obliged to take responsibility for all of the residual errors and infelicities that follow, whilst most of the sense therein must be ascribed to others. These *discussion documents* are in part supported by a University of Cambridge "innovation in teaching" grant to the *Ethics in Mathematics Project*.
[2] http://www.ethics.maths.cam.ac.uk/
[3] We hope!

stimulate some further, and better, consideration by other of the different psychological and social kinds of ethical problems mathematicians may encounter, and how there are whole different *categories* or kinds of ethical awareness and engagement. One of the authors (PB-H) has read Kant, and uses terms such as *categories* with great trepidation; we do *not* mean this in a philosophically deep or laden way, but for the moment in a rather more unsophisticated and naïve manner. We are grappling with the breadth and depth of different ethical issues and social contexts in which the ethically challenged mathematician may find herself, and identifying the different behaviours individuals and communities of mathematicians may have. We are not yet at the stage of worrying about the ontological or epistemological status of ethical concerns in mathematics – there is a whole world of professional philosophers out there who would be better equipped for this sort of discussion.

We do not wish to suggest some strict order, whereby the $n+1^{th}$ level only comes *after* the n^{th} level… although level 0 and level 4 are, we would guess, the top and tail of the categories we have noticed. That we have not introduced a 'level 5' is only because we have yet to observe it, and it is our fervent hope that someone else will find a better way to categorise the states of ethical engagement of professional mathematicians. The constant refrain of the authors is that we hope that some mathematician with more training in sociology, or psychology, or philosophy, will come along and do all of this a lot better.

The central claim we want to suggest is that ethics in mathematics is *not* an on-off binary state; the life experiences, the sociology and the psychology of individuals and communities of mathematicians mean that ethical experiences and reactions to ethical issues will vary, and the extent to which an individual will be willing to react (and *how*) to perceived ethical issues is not a single binary phenomenon. We think this simple observation is original, and the consequences that follow from it grope through much that still remains dark to us; we hope we are exploring some new territory and we think it is worth some discussion.

Ethics in mathematics

As the oldest consistently used scientific tool in Western thinking, mathematics carries perhaps the greatest scientific authority. The 'truth' of mathematics is regarded as the paradigm of what we mean by (scientific) truth. At least, this is the case so long as the reader does not think about the debates in the earlier 20th century about foundations and Gödel's theorems – all of which eviscerates the mathematical meaning of 'truth.' This, however, is very much an internal concern of a small subset of very pure mathematicians; to the general public, the authority of mathematics is pretty much absolute.

Sophisticated mathematics is already ubiquitous in 21st century technology. And, as AI, Big Data, algorithms, cutting edge mathematical physics, mass surveillance, and mathematical modelling in chemistry and biology, become more powerful, this is only going to increase. Even the most ardent purists in number theory or algebra can no longer claim to "just do the mathematics" and "leave the implications to ethicists" as recent revelations about global mass surveillance have underscored the immediate social and political impact of their work. It is now evident that one can wield practically all branches of mathematics in ways that have profound social consequences, both for good and occasionally for ill, and that ethical questions can be raised everywhere in pure and applied mathematics. It seems that professional social responsibility and engagement in mathematics-specific ethical issues is unavoidable for mathematicians; we would

suggest that those who deny this may be abdicating responsibility and power to others in the most dangerous way.

Governments and industry know perfectly well what they are purchasing when they support apparently entirely pure mathematical research and the institutions where it is done. Modern mathematics is the most amazingly powerful tool, and governments and corporations know it all too well. Furthermore, it is not just the mathematics these entities are paying for: they are also supporting the training of more mathematicians – people who know how to wield the power of mathematics, whom they can hire. Mathematicians are one of the most important engines of power in the 21st century, vastly more than they have ever been at any time in the past. Over just a few generations we have been catapulted from a science little different from (and only a little more useful than) philosophical studies to the refined oil that fuels modern high-tech industries and economy.

A hundred years ago, perhaps even seventy years ago, a mathematician might have been excused for thinking that the subject was (at least sometimes) value free, pure, and removed from any ethical questions or issues of social responsibility. *Perhaps*. Since the middle of the 20th century (or even long before) medics, lawyers, biologists, physicists, and people studying social sciences like anthropology or sociology have acknowledged that their work is done in a subject-specific social context with concomitant ethical consequences and issues. They have understood their subject's social responsibilities and become aware of potential ethical issues that practitioners need to know, and to think about. We feel it is hard to imagine that the profession of mathematicians is any different.

But being aware of ethical issues in mathematics is not just a simple consciousness that there exist ethical issues in mathematics and mathematical practice. Simply being intellectually aware that there exist ethical issues in one's professional domain is not the end of it: the mathematician has a responsibility to think about what she can *do* about it, and what exactly that *something* is can vary based on the person, their concerns, their situation, and their willingness to expend effort. In our work on ethics in mathematics, we have identified four *levels* or *kinds* of what might be called ethical consciousness or ethical engagement which we outline next.

Ethics: four levels of involvement

We mean this to be specific to mathematicians, but it is always possible that these levels of engagement might be appropriate for other professions or other cases of ethical engagement. However, our intention for now is to try to understand the different kinds of engagement mathematicians may have with different kinds of ethical issues.

This is still very much work in progress, and we are putting it here as a 'discussion document' as a way of trying to clarify our own thinking and to see if it sparks off any further considerations from others. We do not think that long arguments about precise categories are particularly fruitful (at least, at this early stage in developing the study of ethics in mathematics), but noticing that ethical awareness and ethical engagement occur in quite different ways and places in mathematicians' lives seems a useful way of distinguishing different professional ethical phenomena and different classes of engagement with ethical issues. We have here a list of four levels (well – five because we include a zeroeth non-level) of awareness and engagement in the ethics in mathematics – where the null state is to think that there is no ethics in mathematics, or

that ethical consequences are not the responsibility of the mathematician – hence four levels of ethical involvement.

Level 0: Believing there is no ethics in mathematics

This is where most mathematicians are today. Many feel that mathematics is value-free and that mathematics is inherently extra-ethical, or that mathematics itself does not have ethical implications or consequences. Rather – this line of thinking goes – any ethical issues solely lie with the *user* of the tool their mathematics helps create. Many consciously exclude themselves from the issue by reducing their role to that of the discoverer of mathematics, arguing that "the mathematics was always there anyway."

In our view this is somewhat akin to gun manufacturers and arms merchants arguing "Guns don't kill people; people kill people," and therefore that they have no ethical, moral or legal responsibility in the killing of people using their guns. Legally this is simply not true, of course: the merchants and the manufacturers do have legal responsibilities since in many civilised nations there are laws governing who can and cannot be sold a gun, or what kinds of guns are legal in what jurisdiction, or what properties of guns are accepted for which class of owner.

We have noticed that most mathematicians are implicitly *taught* to be, and think at, this level. They deny that ethical issues are raised by doing mathematics and argue that any ethical consequences that follow from their mathematics do not lie within the proper domain of the working mathematician. They always point to someone else as having responsibility. Of course it is also the case that many *users* of mathematics will deny social responsibility, saying they are only using the tools given to them, and they aren't responsible for unforeseen or unannounced ethical consequences; this is the 'collateral damage' of the mathematicians.

Some mathematicians will obfuscate any ethical content to their mathematical work, saying that "I'm just doing mathematics, just some scribbles on paper … the symbols on the page do not carry ethical weight," whilst others might simply take the attitude that "it's not my problem." The traditional Platonist view held by many mathematicians is that the objects of mathematics are neither physical nor material but have some sort of non-material, abstract or even transcendental existence, that mathematical entities exist in some sort of transcendental *other kind* of reality, and so can harbour no ethical weight in themselves. It is a bit hard to ask a triangle, made of abstract lines of no thickness that only meet at dimensionless points, to have ethical weight in and of itself. Platonism is the standard view of mathematics held (usually implicitly) by working mathematicians, and it is not our intention to argue against it, but only to suggest that no matter how Platonist they may be, it does not obviate their social and ethical responsibilities. Mathematics may well 'exist' in the sense of being a different *kind* or *type* of reality, but it is *done* (discovered, developed, studied, understood, used) by humans in our very material, social world and therefore exists *for us* in some sort of social matrix or social context.

One may try to argue that mathematics, in the completely abstract sense, is devoid of social responsibility and ethical conundrums, because it already exists in some separate transcendental reality and all the mathematician does is explore and make discoveries of entities in that transcendental reality. However, this is beside the point: in human experience, mathematics only exists in so far as humans think about it and study it. The *mathematician* wielding the mathematics is a living, breathing person and certainly is not in that transcendental Platonic reality, and the discoveries of mathematicians are done in the social context of the working

mathematician and therefore become potentially ethical the moment they are made real and non-transcendental in the mathematician's mind. Our work may be abstract, but it is always done by real and concrete people in real and concrete social spaces. This ridiculously obvious point changes – obviates – the Platonic argument that mathematics is extra-ethical.

Level 1: Realising there are ethical issues inherent in mathematics

The first level of ethical awareness is the fundamental understanding that the working mathematician exists and works in a social and political matrix, and that the social matrix can impart some ethical consequences to the mathematics. One always performs mathematics in a social and political context, never in value-free isolation; our professional world is always potentially ethical. Thus, all mathematicians must think about their individual responsibility, as ethical issues may emerge at any time; mathematicians thinking about ethics this level have at least come to the realisation that their work has the potential of having ethical consequences. This is not to say that the mathematics is intrinsically bad, nor that the ethical consequences are necessarily bad. It is to be hoped that most of us work in places or institutions that work for the common good, and that the mathematics we produce in research or which we apply is in some way used for the greater good. But not necessarily: mathematics is an extraordinarily powerful tool and can be used for good, and for purposes that are not so good. The mathematician may not be doing the harm, but he is making the tools that someone else is using for harm.

Such realisation empowers the mathematician to assess, and alter, their actions accordingly. This diligence could be as simple as considering the environmental impact alongside cost and time constraints of some project, rather than producing a mathematical analysis optimising over time and money alone. Mathematics can pose immediate or distant consequences that generally manifest as good, sometimes as not entirely good, and occasionally as downright bad … for appropriate definitions of good and bad. Mathematicians need to be able to consider and weigh such issues, and to be sensitive to the complexities of what might be 'good' or 'bad'. At such an individualistic level, mathematicians modify and adapt their own ethical consciousness and actions, taking the important first step towards a more robust ethical awareness.

Mathematics can change the world – just look around you: how much mathematical research lies behind almost everything around you, somewhere at the back of a causal chain of events? And, in the modern world, this chain is getting shorter and shorter by the day. It seems quite clear that we can ask if the power of mathematics *really* is without ethical content. We feel that this can be illustrated over and over, and these consequences may be immediate and proximate, or very distant in both form and time. Pure mathematics may not find applications for decades or longer, and it is obviously hard for its practitioners to feel the ethical weight of consequences that emerge long after their lifetime. But this is not uniformly the case, and the potential long term non-applicability of some mathematics does not absolve the mathematician of ethical responsibility: not knowing exactly how long it will be before a given mathematical discovery will become 'useful' or have consequences somewhere does not absolve the mathematician of all responsibility. If one part of an individual's mathematical work seems to be genuinely ethically neutral or without any ethical consequences, the next may not. Any temporary suspension of ethical responsibility is not global, it can only be local. Either way, it needs to be thought about, and the mathematicians at this level of ethical engagement are doing such thinking. Just because one piece of mathematical knowledge seems to have no ethical consequences does not mean all mathematics has no ethical consequences; that is no more than a trivial error of logic.

The conclusion may be the ethical neutrality of this or that bitt of mathematics, but our argument is that this should be an individual's *active* conclusion, not a default or passive presumption. Hardy's escape from ethical responsibility and utility (to his evident and profound relief) in the early 1940s was his perception, at the time, of the utter lack of utility of number theory or quantum mechanics[4]. It is hard to imagine two areas of mathematical study that became more ethically laden within a few years of Hardy. What appears today to be so esoteric as to have no ethical weight may turn out to be ethically laden tomorrow. Hardy could not have known in 1940 just how wrong he was, but that is beside the point; a mathematician needs to be in a position to realise that he *might* be wrong. Not knowing in a particular case is not a reason to deny the *need* to consider the possibility, and even the purest of mathematicians at this level can be sensitive to this possibility. After all, a parent cannot totally control the actions of the child they have raised to adolescence, and nor should they be expected to by that point; the child has taken on its own ideas and direction. But it would be irresponsible for the parent to stop caring about the life of the child, or to let it carry on in a wild way; a responsible parent would still attempt to intervene, provide counsel, give guidance, and even perhaps warn others if the adolescent has the obvious potential to inflict harm. Mathematicians at this level, even those working in academic research in supposedly pure areas such as number theory, can at least understand this concept and the consequences that follow from it.

Often a mathematician is working in academe or in industry on projects that involve uses of mathematics that only she can do. This work is funded because somewhere along the line governments or industry have been convinced that there is some ultimate utility to mathematical research; it is probably hard to find any source of funding of mathematics that supports research and development genuinely and uniquely for its abstract beauty. While many mathematicians feel they are primarily motivated by the pursuit of truth and beauty, it is more than likely that this is not the argument that persuades funding bodies to support their mathematics, pay their salaries, or anything else.

A mathematician who thinks there is no ethics in what he does can easily be pressed with the question "So, who pays for the heating of the building you work in?" Whoever is paying for the heating, salaries, and all the rest, is doing it for good reasons, and almost certainly not because they believe humans have a God-given right to study mathematics. Governments do not have a divinely ordained duty to pay for it, and neither are very many private companies so inclined. Everyone who pays for research in mathematics does so for a reason. Academic mathematicians often remark on how lucky they are to be paid to do what they love, and to have the public purse subsidise their pleasure and delight at the purity, truth and beauty of mathematics. We are indeed lucky to be in this position, but governments are doing no such thing.

It is possible that such high-minded motives might have carried social and political weight at some point in the past (*possibly* …), but they no longer do, and there are mathematicians at this level of ethical awareness who have come to this realisation, and who understand that *all* mathematical work can be seen as potentially related to the world in some way. Whom we accept money from potentially affects what we do and how we do it; understanding that influence may help us be cognisant of it, and perhaps resist some of the negative consequences. Even if you are funded by a seemingly well-intentioned benefactor, or even a charity, it still has ethical implications (potentially positive).

[4] Hardy, G., *A mathematician's apology* (1940). See p.39 of the version freely available at: https://www.math.ualberta.ca/mss/misc/

Quite apart from research, much of a professional or academic's actions will have ethical consequences that are specific to the mathematical profession. Academic mathematicians have additional professional duties that may involve deeply ethical decisions, including writing letters of reference, refereeing journal articles, advising students about careers, and sitting on appointment committees. Consider the following question: do you counsel students to look at (or avoid) jobs in certain areas, such as banking, finance, marketing, the military, or state surveillance organisations? Joining the teams of mathematics working at Google's DeepMind or on Facebook's commercialisation of user data are entirely legitimate places for a mathematician to work. Do you encourage students to work for Google? Facebook? Some local AI project? Would you encourage a student to work for some new manifestation of Cambridge Analytica? After all, there are already many other organisations doing much the same sort of thing as Cambridge Analytica, and there will be more, unless governments suddenly find the will and the ability to regulate this sector (which we have a feeling is unlikely, at). Would you write a letter of reference for a student that wanted to go and work on the North Korean nuclear weaponry project? Or to work for a Nazi party? There are almost certainly lines that you would not cross, and that means you are making ethical decisions; willingly or not, academic mathematicians *are* engaged in ethical decisions. But more importantly: only a mathematician would have the grasp of what the good – or harm – that a mathematician might do working in such places.

So, we observe that this first level of ethical consciousness is to be aware that it exists, and to be mindful of, and modify, one's individual actions accordingly, in a (mostly) individualistic and passive way; asking an extra question here, thinking harder about potential drawbacks there, etc. It is to realise that mathematics is done in a social matrix for reasons, and that mathematics can have consequences – generally good, sometimes not-so-good, and occasionally downright bad. At least for some definitions of good and bad, and the ethically aware practitioner needs to be able to consider what she means by 'good' and 'bad.' To us, this seems to be fairly obvious and an easy conclusion to accept.

The first stage of ethical consciousness is just that: to be conscious of the existence of potential ethical issues that you may need to think about, talk about, or respond to. Once you have realised that something you might be doing may not be ethical, what are you going to do about it? This is a *second* level of ethical 'consciousness' that requires more than just awareness; the discussion is not merely academic insofar as how mathematicians view and shape their everyday actions.

Level 2: Doing something: speaking out to other mathematicians.

Now comes the question: "what can a mathematician *actively* do?" This is where the going starts to get tough, and where these discussions may become uncomfortable. Openly and vocally responding to something ethically undesirable may come at a cost to the practitioner. *May*, but not *necessarily* – not all ethical consequences will be bad or undesirable, and even those may be resolved at little or no cost. But some will not be so easy, and this may involve the mathematician in some hard decisions. For example: a mathematician may object to carrying out a particular task, and subsequently get fired (although we would argue that given the scarcity of mathematicians, and the fact that the objection of one may propagate through that particular workplace and lead to others objecting, as in the case of the Google engineer strike[5]). Ideally, it

[5] Over 3100 Google engineers signed an open letter protesting the company's involvement in a Pentagon program that uses artificial intelligence to interpret video imagery and could be used to improve the

would be better *not* to end up in a situation where such a decision is necessary, and to have the foresight to realise that a particular employer may be more likely to make such a request, or that a particular task may eventually put the mathematician in an ethical conundrum. It is to be hoped that the ethically aware mathematician will have a general idea what is going on and simply not get into a position where management says, "do this or I will fire you."

In the case where a mathematician knows something bad is going to result from some work, they may feel obliged to do it anyway, given the pressures of career progression, mortgage repayments, school fees, etc. Such considerations may limit an individual's personal freedom to act as they like. We cannot ordain that all people act perfectly and ethically in all situations. But that is not to deny the analysis or what an optimal response would be – and living in an imperfect world may oblige someone to do something they do not want to do. One way or another we have all probably been in that situation. It may happen, but we hope not without some recognition of ethical responsibility and some regret. Not everything is easy or perfect in the world, but that does not prevent us from trying as best we can.

We want to be careful here and try to avoid either preaching or making recommendations as to what an individual should do, and we want to try as hard as we can to avoid taking a political stance; that is for the individual mathematician to decide, not us. Sadly, there are no algorithmic rules for dealing with ethical problems in mathematics, and there are no universally agreed ethical standards that would make these problems easy to resolve; no simple equation balances costs and benefits. We, the authors of this paper, have been asked countless times by students for the *rules* of ethical behaviour, and for an *axiomatic* ethical system so that ethical decisions can be made deductively, without any personal involvement on the part of the mathematician. Our response to them is simple: life is not that straightforward.

The second of these four levels may involve mathematicians actively speaking out to other mathematicians, raising awareness of ethical issues among their peers. Individual mathematicians may realise not only that the ethics of *their* work must be carefully considered, but also that these issues are important enough to warrant further discussion among *others*. That is, recognising the ethical issues in the mathematical work of others, and trying to inform them about it. They might precipitate unified action among their colleagues and locally bring about a collective ethical awareness and approach. Or they might write an article or paper about ethics for their community, as we have done here.

This is an important step inside a community that otherwise tends to be reluctant or unwilling to acknowledge that its activities have ethical consequences just as every other profession has. It can be done in many different ways. It could be workplace-specific: a mathematician raising ethical concerns with management and/or other colleagues in a place of work, pointing out an issue or systematic problem that needs to be addressed at a decision-making or managerial level, rather than just at a mere technical level; we saw an example of this with the Google engineer strike. It could be issue-specific: a mathematician speaking out to the mathematical community about a particular topic of great concern, such as Prof. Martin Hellman[6] who is actively speaking

targeting of drone strikes: https://www.nytimes.com/2018/04/04/technology/google-letter-ceo-pentagon-project.html . The letter can be found here: https://static01.nyt.com/files/2018/technology/googleletter.pdf. Their efforts led to a tangible change in the policy of the company; Google chose not to renew the contract with the Pentagon: https://www.bbc.co.uk/news/business-44341490 .

[6] Turing Prize recipient, and co-inventor of the *Diffie-Hellman key exchange*; one of the earliest public key exchange systems.

out and raising awareness about a potential fallacy/inconsistency in the maxim of Mutually Assured Destruction for nuclear war, which comes from a mathematical argument built on the possibly questionable assumption of completely rational actors. It could even be a community-wide effort: academics – even the purest of pure mathematicians – are heavily involved in teaching, and may take it upon themselves (as we have) to incorporate the teaching of ethical and social responsibility to their mathematics students as part of a more robust 'professional training'. This 'speaking out' to the mathematical community can come in different ways, focusing on different things, and on different scales. But the recurring theme is clear: certain mathematicians have reached a sufficient level of ethical awareness that they then take it upon themselves to inform, and perhaps persuade, others in their community.

Mathematicians at this level need to be aware of some of the mechanisms for speaking out that are available to them, milder than potentially life-changing acts such as whistle-blowing that may lead to legal problems, prison, exile, or other huge personal consequences.[7] It may involve discussing things amongst other mathematicians or employees, talking to managers, or organising in a more-or-less political way, or speaking out in public, or speaking to outside activists. The avenues that are available to a mathematician to engage with the ethical consequences of their work are many. One of the things those of us who are thinking about ethics in mathematics need to do is start to discuss – and talk publicly about – the different ways in which mathematicians can act. Every mathematician, regardless of their current level of ethical engagement, will need encouragement and guidance from the rest of the community, and we need to learn what to do, and teach what to do. This is one of the first duties of those who are concerned about ethical issues in modern mathematics: we need to discuss and teach what can be done, and *not* think we can prescribe normative behaviour.

Whatever it is that mathematicians might do, we need to realise that any discussion of this requires an analysis of how mathematicians behave. We believe this is an extremely important condition, and one that mathematicians generally do not want to take into account. Mathematicians are a self-selected and well-defined group of people: you know who you would call a mathematician and who you would not. We need to understand better what the natural, inherent characteristics of the members of the community, or of the profession, are. How do those personality and intellectual characteristics function in various places in the social matrix of academia or as part of the mathematical engine room of a corporation? These may be generalisations, and we may well be over-generalising, but it may still make for hard reading for some[8]. The analysis of the natural behaviour of mathematicians is beneficial even if over-generalised because it points to the strengths and weaknesses of a mathematician's reaction (and the behaviour of the community of mathematicians) to confronting ethical consequences that she may not like.

Many professional, working mathematicians are exactly the publicity-shy people who are not going to enjoy the limelight, and do not enjoy controversy or public confrontation. They may be

[7] The whistle-blowing actions of members of various intelligence communities (such as Edward Snowden, or Chelsea Manning, or William Binney and Thomas Drake) in revealing mass surveillance programmes demonstrated a very deep ethical awareness of their work and its impact, but came at huge personal cost.

[8] One of us (MC) has had several meetings with the computer science, IT development, and hacker communities, and has spoken to them about such things as ethics. He has been met with astonishment from them about the perspective he has shared on how our mathematics students and colleagues behave when faced with one ethical question or another; a perspective that other mathematicians present at these events or gatherings have confirmed. There really *are* professional stereotypes that have some validity here.

vicious, harsh, and ruthless when arguing about mathematics over email, without face-to-face social contact, but neither the heated debate nor the social subtleties of the committee room or the public podium are the most comfortable place for them. Moreover, many mathematicians are not culturally well trained or well prepared to engage in public argument of an ethical, political, indeterminate nature and the reader might note that nothing in our training as mathematicians addresses this. Of course, this is a generalisation, but it is probably a reasonable generalisation of the norms of mathematicians' behaviour.

Mathematicians in industry are often managed, and sometimes even exploited, by employers and managers, who know exactly how to work with mathematicians and how to accommodate them. They give them the work environment that mathematicians prefer, and in which the mathematical work will be done best (and therefore successfully … profitably … for the corporation). There are circumstances where mathematicians work in environments carefully constructed by clever, socially manipulative managers – often very successful people who are, quietly or loudly, psychopaths who know exactly how get what they need from other people. They are successful managers exactly because they know how to exploit others for the benefit of the corporation, and part of the environment they construct is one where they can keep mathematicians (for example) isolated from the consequences of their work. These are work environments that mitigate against mathematicians developing ethical consciousness about what they are doing, let alone doing anything about their ethical qualms.

And here is the rub: if we give mathematicians an education and professional training with *no* hint of ethical sensibilities, without *any* exposure to the social context in which commercial and industrial mathematics is done, if we teach mathematics – to future professionals in an ethically passive way, them we are sending out students – some of whom might have less social awareness compared to other graduates – *without the wherewithal to find appropriate ways of raising ethical concerns in the workplace*. If, on the other hand, mathematicians in industry have some awareness – perhaps through training – of the need to talk with each other about these issues and to find ways to react and speak out, it may well lead to outcomes such as collective action amongst a group of mathematicians, which may ultimately protect them against such manipulation and exploitation.

Level 3: Taking a seat at the tables of power.

The first two kinds of ethical engagement that we have discussed involve mathematicians stepping out of their emotional and professional comfort zone and doing something different, but almost always exclusively within the confines of the job or position they already hold. The third level is more complex. Our argument is that once mathematicians are ethically engaged and realise that they need to act, to find appropriate ways to react to ethical concerns, they need to be able to *move* from where they currently sit (metaphorically speaking), and get a seat at the tables of power. While the previous two levels could be summarised as "do something different, from within the same place" this level is closer to "move to a new place, and do something different."

However mathematicians might want to deal with the ethical questions they may come to realise their work entails, one important way in which they can begin to deal with ethical issues is what we are going to call 'getting a seat at the table' – that is, to be part of the management and decision-making processes that define the problems that mathematicians work on, or decide what is done with the results of mathematics. Of course, this is exactly what mathematicians do not want to do:

we are not trained to do it, and it takes us away from mathematics and launches us into an entirely different domain.

What happens frequently in industries where high level mathematics is done is that the mathematicians are in effect left alone, in peace and quiet in the cellars of the company (again, metaphorically speaking) in socially uncomplicated settings and given mathematical tasks perhaps without any context. They can get on with what they like to do, and what mathematicians are so pre-eminently and uniquely good at doing: solving interesting, fun, hard mathematical problems. What they are not good at, and what they are not naturally attuned to doing, and what we do not train them to do (and is just a distraction), is work with others at the management level, at the level of commercial and policy decisions, discussing their mathematics with non-mathematicians (and discussing the perhaps unseen power and consequences of their mathematical tools), making management decisions about what is done with the mathematics or what mathematics should be done, or what the mathematical results actually mean.

The stereotypical mathematician precisely eschews all those socially difficult, under-determined and emotionally complex sorts of loci of power and power exchange. Mathematicians are good at mathematics, but often not so good at committees and positions of power where social negotiation and nuance rule. Computer scientists, lawyers, even medics are taught some semblance of business or management skills; top ranking universities do not tend to teach mathematics undergraduates these skills. Mathematicians are good at conflict with each other about mathematics, but they are awful at social situations of power and conflict. Not all, of course, and not always, but generally speaking, the first-rate mathematicians that big corporations hire to do their complex maths, or that small cutting-edge start-ups use to do their sophisticated high-tech mathematics are just not culturally suited to the committee room with non-mathematicians engaged in the manipulation and exercise of power. But that is exactly where mathematicians need to be: at the table, engaged in the discussions of management, and engaged in the exercise of power, in academe and in industry. Because *that* is one of the prime places where the mathematically able and ethically literate person can actually make ethical decisions or direct others to take into consideration the ethical issues and consequences that may arise from mathematics.

The most important point here is the more-or-less unseen or unstated consequences of mathematical tools: it is often the case that only a mathematician will understand the meaning, validity, extendibility, reliability, or array of consequences of some mathematics (see also level 4b below). But the cold, separable results may be used and misused by others with scant understanding of the mathematics and therefore of the instruments they are using. At the table where management and power lie, the ability to understand the mathematics in a deeper way may not be present; being at that table gives a participating mathematician the potential to influence the ethical consequences of what is done with the maths. There are mathematicians at this level have made the same realisation, and are taking steps to fills these seats. A high-level example of this is Cedric Villani, the Fields Medallist who sits in the French parliament, who recently prepared a thorough and remarkably persuasive report[9] for the French government on how to prepare France for the arrival of AI in mainstream society; the particular strength of this report is that its authors have a deep grasp of the technology and the mathematics behind AI, and that gives them a particular political authority.

[9] Villani, C., et al., (2018). *For a Meaningful Artificial Intelligence*, (2018). Retrieved from https://www.aiforhumanity.fr/pdfs/MissionVillani_Report_ENG-VF.pdf

The irony, or maybe the tragedy of this state of affairs is that one of the groups of people who have the most effective power in the world are, as a group, the most socially and psychologically under-prepared and professionally unwilling and untrained to exercise or manage that power. Mathematicians are notoriously unwilling or unable to explain their mathematics to the non-mathematician, and as a result there is often a vast disjunction between what a mathematician knows her maths can do, what it means, how reliable or meaningful or probabilistic its results are … and what the non-mathematical or semi-mathematical public believe the mathematics means. With many of the algorithms in use in the world, be they in technological, financial or general social applications, there is a completely mistaken trust and reliance given by the general public, or other professionals such as lawyers, judges, or policy advisers, which mathematical practitioners know is mistaken or exaggerated. Mathematicians need to be in corporate, managerial, government, and institutional positions that will ensure that mathematics is not given an exaggerated power, or trusted in circumstances where it does not have the certainty that lay people generally attribute to it, or where it is just used completely inappropriately, or even just downright erroneously.

But this is not easy for most. Mathematicians often need to learn the specific skills required to work with politicians, corporate management, and other non-scientists. These include engaging in policy discussions, establishing and rationalising the objectives of their mathematical work, and learning ways to communicate to a non-mathematical audience potential limitations, overreach, and possible drawbacks. Engineers and computer scientists are taught this throughout their undergraduate training, but such lessons are rarely communicated even implicitly to mathematicians. Many mathematicians in advancing industry careers find themselves in positions that require these abilities, often unexpectedly. Mathematics is becoming an increasingly powerful social tool, and seeing its creators hiding behind formulae and retrospectively apologising is an insufficient response and solves no problems. Mark Zuckerberg's woeful performance in front of the US Senate earlier in 2018 is a classic example of the *politically* inadequate (Zuckerberg) meeting the *technically* inadequate (US Senators) … and users around the world are likely to pay a long-term social cost we have not even begun to understand.

However it is that Facebook is managed (and in some sense, it must be managed *brilliantly* given its short-term success), it is clear that until very recently they have had little or no discussion of the downside, the potential ethical problems their technical tools have created. They have been near to catastrophically unprepared for the ethical questions raised by the power of their technical tools. The technical people – often mathematicians, some may even be graduates of your university – have (*i*) simply not been in positions of power, influence and responsibility *and* (*ii*) not been given the job of considering or explaining the larger ethical implications of the tools Facebook has and how it uses them. These two conditions together are likely to be an excellent way to create systems that can do individuals or society extraordinary damage. In unregulated industries this is beyond dangerous, and it is far from clear that modern, Western regulators have any grasp of how they might even begin to regulate such industries.

If, as mathematicians, we want to take credit for the positive impact of our output, we should also be able to properly contextualise our work and a range of foreseeable – and distantly possible – implications, and engage in apparently non-mathematical debates and policy analysis based on the mathematics. To do this, mathematicians need to be in the corporate, institutional, or political locations where these issues are discussed and where policy decisions are made. This is the level

of management and the kind of social interaction that many mathematicians simply hate; this is a distraction from doing what they do best: mathematics. To many mathematics this would be a form of torture, whereas to most of the outside world this would seem to be little more than a natural extension of their mathematical work. So much mathematics is so far removed from the language of everyday discourse that only mathematics can do this effectively, and a seat at the tables of power is about the last thing that most mathematicians want.

Fortunately, some mathematicians have already come to this conclusion, and occasionally are doing something at this level.

Level 4: Calling out the bad mathematics of others

Our fourth level of ethical engagement is the responsibility that falls on mathematicians to call out the bad mathematics of *others* by proactively seeking out, learning about, and acting upon, instances where mathematics is wrong or has 'gone wrong' – in organisations and institutions totally unrelated to them, outside their professional or social domain. Only mathematicians can provide a deep perspective, analysis, and understanding of mathematics, and we need to be willing to speak out and give witness to the ways mathematics is used or the limitations or correctness of some of the things mathematics is asked to do. But this witness needs to be more than just to other mathematicians; it needs to be to a wider public.[10] How often has a mathematician looked at some algorithm or heard about the operations or the decisions of some system and just quietly said to themselves 'ridiculous' or 'rubbish,' or just noticed that the result does not have the significance or meaning that is attributed to it? Only a mathematician can understand and judge much of the mathematics that is in the public domain today, be it obvious, or subtle.

However, bad mathematics comes in two distinct forms, and the circumstances and ways in which mathematicians may react to it may differ.

First, where non-mathematicians misuse mathematics (deliberately or through incompetence or rank ignorance); where the non-mathematically trained use mathematics incorrectly or inappropriately, claiming results that are *not mathematically true* (although they give the appearance of being correct to the uninitiated) and

Second, where the mathematically trained use mathematics but in inappropriate or incorrect ways, or for what is *prima facia* unethical purposes; where the technicalities – impenetrable to the general public – hide errors or bias, or malign intent. There are also the cases of sophisticated mathematics that is simply an *inappropriate* and a *wrong* use of mathematics, where the calculations may be formally correct but the results do not have the claimed significance.

4(a) Calling out non-mathematicians using maths unethically

In the **first case** there are plenty of examples of people who try and *use* mathematics, but who, ultimately, are not mathematicians or mathematically competent. Thus, they are perfectly likely

[10] We have a recent case of this in the UK: Dr. Hannah Fry is a young mathematician at UCL and has become a media figure on YouTube and TV, talking about mathematics and mathematical machines, algorithms, and AI, and "how to be human in the age of the machine." She is young, well-presented, lucid, clear, informal, talks without being patronising, smiles a lot and is enthusiastic, and can expound on hard ideas to a general audience; she has legitimacy and authority. Obviously she is going to be a media star … although heretofore she has not addressed in depth more controversial ethical questions in her talks and popular writing.

to misunderstand, misapply, and misuse such tools without knowing it, and therefore claim mathematical results that are simply not true. Observing such errors may seem trivial to most mathematicians, but the consequences of these errors can be serious.

One dramatic case of this is the catastrophic misuse of statistics by a paediatrician in the trial of Sally Clark[11], which the Royal Statistical Society subsequently – two years later – called out. Just because *professional, trained* mathematicians can see some piece of mathematics as being obviously wrong, does not mean that relevant people outside the profession are capable of making the same observations (in the case of Sally Clark: the defence lawyer, the judge, and the jury). This can often slip through the cracks as a by-product of the indifference of many mathematicians who simply do not bother to take such actions, and the general public that has been persuaded, even trained, to have a nearly unlimited and unquestioning faith in mathematics. Statistics is the part of mathematics where this is the most obvious – and any reasonably professionally qualified statistician can tell of a large number of statistical horrors where statistics and statistical calculations are simply wrong. The egregious misuse of statistics is well known to mathematicians, but one has to ask why is not the rest of mathematics, pure and applied, not seen in the same light? Statistical claims are not the only mathematics one sees in newspapers, for example, and just as journalistic statistics are often wrong or misleading, why should we expect that other uses of mathematics by journalists (or others) are not?

It is a commonplace – but a true commonplace – that mathematics has an ineluctable truth in its arguments, providing absolute truth in an absolute defence of an idea. But since mathematics is the paradigm of what we mean by *truth*, surely this is perfectly acceptable? Obviously the answer is *no*: as the intrinsic truth present in mathematics does not translate to being *necessarily* usefully true, and more importantly having appropriate *meaning* in every conceivable context or application. In many areas, this will be found out almost immediately. If someone uses an inappropriate or overly simple mathematical approach to building a bridge, we fairly quickly see it collapse, and soon after know whom to blame (in this case, who misused some mathematics). But in some areas, mathematics can be misused in a way that is not immediately obvious, like widely-used prison sentencing algorithms such as COMPAS. Much touted as providing sure conclusions (and appropriate prison sentences) from objective mathematical calculations, they actually use baby maths *in a really naïve way*. In the case of COMPAS, it is based on nothing more than a weighted sum, and as it turns out the weighting parameters were obtained in a completely unreliable, un-rigorous and un-scientific manner.[12] The algorithm has none of its claimed mathematical objectivity.

[11] Two of Clark's babies died of cot death separately, each soon after birth, and a paediatrician 'demonstrated' in court that the likelihood of this happening naturally was minuscule, backing up the accusation that she must have murdered each baby; on the basis of this she was convicted and jailed for the two murders of her babies. After the RSS showed the statistical arguments used by the paediatrician were completely wrong and bogus, she was released and her conviction quashed. However, it took the RSS took two years to issue their statement showing the errors in the original trial, and Clark, completely innocent, served three devastating years in jail. She died four years after her release due to alcohol abuse and poisoning after prolonged and serious depression. Her trial and wrongful conviction ruined her life and effectively killed her … that is to say, the erroneous use of statistics caused her early and untimely death.

[12] The weights were computed by giving a survey to existing prisoners in that geographical area (state, county, etc), asking them to fill it out honestly, and then observing various correlations in the data. It is not a far-fetched idea to think that existing prisoners might not be a reliable source of data, or that the observed correlations do not actually amount to *causation* and thus may reduce the reliability of the eventual algorithm. See: http://www.northpointeinc.com/files/technical_documents/FieldGuide2_081412.pdf

Mathematics is often revered in society as some sort of black art, and it takes a professional mathematical practitioner to identify, and critique, misused mathematics. It is one thing to apply correctly a formula, but only a master of the profession can understand exactly what a formula actually means and in what ways it may be relevant and not relevant; unintended implications and consequences in the application of a given formula require more than simplistic analysis. Applying *the correct formula,* assuming there even is one, still leaves the question of where bias, choices and subjectivity may enter without the end-users knowing. Calling out these issues requires a different sort of commitment from mathematicians.

So why are mathematicians generally so slow and reticent to address the *dumb* misuse of mathematics, if at all? Simply put, they don't see it as *mathematics*, they see it as little better than some form of false use of mathematics like numerology; the mathematically-untrained attributing powers to mathematics in a completely ridiculous and mathematically meaningless way. Why did it take the Royal Statistical Society two years to call out the incorrect use of statistics in the Sally Clark case? Apart from institutional inertia and the time it takes a report to get through institutional committees, it just was not that important an issue to most practising statisticians. Mathematicians often are insensitive to the mathematical ignorance and mathematical faith of the wider public. To many mathematicians, any 'average person' (which often implicitly means 'with a first-class degree in mathematics') would *obviously* see that an error or misuse has occurred. It is *so* stupid that it is not even worth their effort to point it out. Nobody in the mathematical community would acknowledge it as a worthwhile observation; who cares if a mathematician identifies a trivial mathematical error made by a pædiatrician? One is not going to get a paper published in *Annals* pointing out incompetent and obviously erroneous statistics.

This sort of misuse of mathematics falls through the cracks. It is not sophisticated mathematics, so trained, professional mathematicians in academe and in industry will tend to ignore it, and with good reason from an academic or professional perspective: there is no individual career or professional community recognition likely, nor likely potential for a raise or promotion, or in general even a pat on the back. Indeed, such an action would generally be look on as simply a waste of time; time that could otherwise be spent doing 'serious' (i.e., abstract, or profitable) mathematics.

Yet it might be the case that the mathematics is hidden (as in COMPAS) or is sufficiently sophisticated that it still earns the Teflon shield of 'real' mathematics to the uninitiated. It appears invincible. It is generally *unlikely* that many in the lay public will be able to reach the level of mathematical understanding needed to scrutinise this faux maths. There have been mathematically trained observers who have examined or called out such mathematics for what it might be: without authority or just glorified numerology. The ethically minded mathematician may be in a position to point out *how* such misuse might be harmful, perhaps even *before* such harm comes to pass. This is a public-facing ethical engagement that most mathematicians are going to find challenging, or without professional reward, and socially difficult.

4(b) Calling out mathematicians using maths unethically

In the **second case** of 'calling out bad mathematics', professional mathematicians find themselves in a more difficult situation because the 'bad mathematics' may be a great deal more technically deep, and above all is done by the mathematically competent. This is a social, professional, and ethical stand that is considerably more difficult for a mathematician. She may be faced with another trained mathematician's inappropriate use of mathematics, by (more-or-less hidden) uses of sophisticated mathematics to give conclusions an excessive authority, or by the use of

mathematical tools beyond the grasp of the general public that can (deliberately or not) cause harm and exploit others. The difficulty in this sort of ethical engagement lies in cases when unethical uses or consequences of mathematics are caused by mathematicians doing 'real' maths, and only mathematicians can call out this level of mathematical work. A public engagement with ethical and unethical use of mathematics at this level does not sit easily with the community norms of mathematicians' behaviour towards the work other mathematicians.

The work of mathematicians is extremely technical and deep. It often takes years (or decades) of persistent training in a mathematical discipline or sub-discipline to reach a sufficient level so as to be able to understand, let alone critique, the mathematical work of others. This means membership of, and commitment to, the mathematical community. Being professionally trained in mathematics, we have a much greater ability to identify and isolate the uses and misuses of mathematics in society. We can see where it comes from, what it has done, and where it is going. Call it a 'sixth sense' if you will, but really it is just a by-product of years and decades of training; we can detect mathematics at work, in places that the general public has no idea about (for many, what goes on unseen is little different from magic). A lot of the work of mathematicians remains quite hidden from the public; just imagine the mathematician-hours there are embedded in the technology of every smartphone. All the public see is the outcome (a bridge was built, their loan was rejected, their smartphone app suggested dating someone) not the mathematics behind it, and do not even realise that mathematics is being used in these places.

The point here is that if mathematicians *are* doing something irresponsible or unethical, then it is often *only* other mathematicians who can identify the source of the problem, and see that the mathematical work behind it actually played a substantial part in enabling something harmful. It may very well be that the public do not even *realise* that there is a problem, just as for many decades no-one realised smoking was bad for them; making the causal link between such things is hard. To put it simplistically: only a mathematician can out another mathematician.

Members of any profession have the responsibility to hold their work – and the work of their colleagues – to high standards. Like statisticians, engineers, and doctors, ethically engaged mathematicians must articulate their own form of professional standards in academia, industry, and more generally in society. Some mathematicians are already questioning the validity and fairness of various decision-making algorithms or identifying the potential dangers of artificial intelligence (AI), bringing such risks into public consciousness and proposing workable solutions.[13] The false claims of the objectivity of algorithms in general (and many specific cases) has been the subject of some recent public comment by some mathematicians,[14] although the technical level of the analysis is still very low.

An excellent example of a mathematician calling out other mathematicians is the work done by Paul-Olivier Dehaye in helping to uncover and expose the activities of Cambridge Analytica (CA) in manipulating British and American elections. Journalists were already investigating what CA was doing, but had little grasp of what exactly was being done with the data that had been obtained (possibly illegally, certainly unethically); the malign and unethical (and undemocratic)

[13] There is a large and rapidly increasing concern and literature on the ethical dangers inherent in AI, 'Big Data' and similar subjects.

[14] Cathy O'Neil's *Weapons of math destruction* is the obvious example, as is Safiya Noble's *Algorithms of Oppression* (although neither are a deep investigation of the technical mathematics of the algorithms). The secondary literature on the danger of the claims of algorithms is expanding quickly, and becoming more technically focussed than the more journalistic approach of O'Neil and Noble. With notable exceptions little of it is aimed at the general educated public.

project and backers of CA was clear, but not the details of how they did it. One of the managers of CA – Alexander Nix – was repeatedly quoted as saying he enjoyed dealing with reporters because he knew that they had no technical grasp of the mathematical technology that CA was using so he could pull the wool over their eyes whenever and however he wanted. What was needed was someone mathematically trained enough to get behind the misinformation and misdirection of Nix and his colleagues.

It took Dehaye to drill down to the level of the mathematics to unpack the mathematical technology that CA used, and therefore precisely how their psychometrics and information manipulation (that is, targeted misinformation) worked; the journalists' *suspicions* of the unethical basis and unethical behaviour of CA was only confirmed and detailed when a technical grasp of how their manipulative and targeted mis-informative advertising worked. Dehaye saw that mathematical practice was being put to bad use, and he applied his expert knowledge to reveal that great harm was being done to democratic processes in the form of targeted advertising and deliberately targeted misinformation; the targeting being extremely effective as a result of the sophistication of the mathematics behind it.

It is interesting to note that mathematicians critiquing mathematicians is something that occurs quite often, but in a completely community-internal way without any particular public engagement. Mathematicians are quick to critique the mathematical work of others and find errors or inadequacies in published work, or generalise the results of others, or observe that their published results have appeared previously in some form. Within the community of mathematicians there is an extremely well established and understood mechanism of identifying and communicating critiques of other mathematician's work. Mathematicians are good at calling out the work of other mathematicians, but only within the strict confines of their own community.[15]

Why is this not done *outside* of research mathematics? Why do so few mathematicians take the time, or care enough, to denounce bad mathematics done either by the technically less competent (4(a)) or by the competent (4(b))? There are **two** obvious barriers and these are issues that the mathematical community could choose to address.

In the **first** instance, it is not clear or obvious to most mathematics *to whom* one would call out bad mathematics. A letter to your local parliamentary (or legislative) representative (who probably has done no mathematics since they were 16 and has the traditional reaction to anything mathematical: "it was my worst subject at school")? A Letter to the Editor? A denouncement in your personal blog, read by three family members, six friends and 12 colleagues (including ten current or former students)? Your line manager at work? This is an obvious but serious issue: mathematicians are congenitally poorly connected to the world of power and publicity – see level 3 above! – and knowing where to denounce bad mathematics, finding the right audience, is a skill and the result of a kind of networking that mathematicians generally do not have. Generally we would not even know where to begin to learn this skill. Lawyers are better connected, medics and engineers have professional bodies that *expect* to hear, understand, and to take on these sorts of issues, and their professional organisations able to begin to interact with public policy. The collective organisations of mathematics (IMU, LMS, AMS, EUMS) are hardly such bodies: they do other things that suit the needs of academic (less so for industrial or financial or government) mathematics, and they do those jobs extremely well. But they are not generally

[15] And even within the community, the muted response to Sir Michael Atiyah's (Fields medallist, past President of the Royal Society) purported proof of the Riemann Hypothesis shows an interesting restraint.

campaigning bodies and are not structured for this sort of purpose; after all, they reflect their mathematical constituency.

So knowing *to whom* the mathematician should denounce bad mathematics is no small issue. It took the Royal Statistical Society *two years* to call out the bad statistics in the case of Sally Clark (and the RSS is a well-known, ethically active and respected profession organisation well aware of the damage of misused statistics!) and there were lawyers and a Court to shout at. Olivier Dehaye was working with outstanding investigative journalists at the Guardian newspaper. Prof. Sir Timothy Gowers is a Field Medallist with a very widely and respected blog[16]. But what about the rest of the mathematical community?

This is a large and pressing issue that needs to be discussed further; as these *discussion papers* grow it is one that we will need to come back to for much further discussion. This is an important issues that the reader is enjoined to consider further;[17] this is another domain where those concerned about ethics in mathematics can actually *do* something useful and concrete. From the beginning it has been our argument that it is important that we find ways of teaching ethics relevant and specific to mathematicians to our students, but if we are to encourage a wider professional engagement in mathematics-specific issues then the community needs to learn and disseminate more than just ethical sensitivities: we need social skills, networking and a knowledge of the corridors of power that will allow the mathematicians' ethical concerns to find a public voice. Lawyers, engineers, medics and other professions know how to do this; there is much mathematicians have to learn from our neighbours.

The **second** barrier to mathematicians calling out bad mathematics is one of professional incentives. People are usually driven by an assortment of incentives and at present there are *no* incentives for a professional (or semi-professional) mathematician in academe or in industry to direct their efforts and energy to identifying the misuse or misapplication of mathematics *outside* the profession. You can get a paper out of proving another paper is wrong; but it is unlikely one can get a mathematically respected paper out of arguing that some technical mathematics can be used to cause harm. Nor will you likely get paid to do it, or any other obvious immediate tangible benefit (from within the mathematical community). Not only is there no obvious entity you can go to and report such misuse, as those who care probably will not understand what they are being told, and those who understand the technical analysis will probably not care, but how many Faculties of Mathematics have an ethics board? Thus, mathematicians face a strangely high wall when it comes to speaking out about the unethical actions of other mathematicians. No community has been built, and no incentive structure formed, to promote this. Some mathematicians may do it of their own accord, but given the limited grasp and interaction with ethical issues by the community of mathematicians, such people are rare.

[16] Professor Sir Timothy Gowers writes a widely read and respected blog, and has addressed issues in the ethics of mathematics publishing (particularly in high cost paper journals that do not have open access digital versions) and has advocated a boycott of one very high-cost not-open-access publisher; he has also recently written about a rather public spat amongst some mathematicians about some rather politicised application of mathematics where he 'called out' the poor quality of the mathematics that was at the basis of the spat (trying not to take sides in the argument itself, which had become to some extent an argument laced with political positions). His denunciation of the mathematics is significant given both his stature as a mathematician and his conclusions about its appropriateness.

[17] Forgive us for giving the reader homework to do …

It should not be a hard argument or claim to make that the highest level of ethical practice is taking responsibility not just of one's own actions, but also of the actions of all of those in the profession. There are mathematicians who will come to the realisation that they must do it because no one else will, nor *can*. The challenge is to go beyond speaking out in a way that only other mathematicians can understand; it involves denouncing mathematics and raising ethical issues stemming from technical mathematics in a public forum and in a language the non-mathematician can understand. This gives such a larger audience opportunity and space for a response. This may be political, regulatory, judicial, or social; there are many tools that society has available to respond to unethical phenomena, but there will never be an effective social response when the origins of these issues are opaque to the general public.

Conclusion

The conclusions we would like to draw at the end of this discussion paper are several: that the state of consciousness of ethical and social responsibility amongst mathematics and mathematical practitioners is – obviously – very much in its infancy, and we all have a lot more to learn and a lot more professional and philosophical introspection to do as we come to terms with the new 'post-modern' immediacy and social, political importance of mathematics. Mathematics-specific ethical issues, and the various social locations where ethical issues present themselves to mathematicians in industry and academe will need a generational effort amongst mathematicians to master. We are not going to solve this problem by next summer, this much is clear.

There is no doubt we have much to learn from other 'new' fields where mathematically minded technical people are coming face to face and responding to *their* subject-specific ethical issues – AI, Big Data, computer scientists in general and other 'fashionable' new technical disciplines are much further down this road.[18] We note that mathematicians often enter these areas upon completing their mathematical training (and without the ethical training that computer scientists and engineers often receive). We have much to learn from lawyers and those who have attempted to bring 20th century regulation to the 21st century digital realm – with extremely patchy success, to be polite. And no doubt in time mathematicians will gain enough reflective skills that we will be able to learn from philosophers. However, we feel that is a long time away, for the simple reason that philosophers have been talking about ethics in general and abstract ways for two-and-a-half thousand years, generally with no mathematical training or interests, and their language, conceptual and analytical apparatus, and the depth of their discussions goes far beyond what mathematicians need or can appreciate at the moment, and is generally not relevant to what we need to think about today. Discussions about ethics in mathematics are new, and are not yet at anything like the level of a standard academic introduction to ethics, let alone what established philosophers talk about.

Practicing ethics in mathematics is not binary, and mathematicians must come to understand various levels of engagement and ethical consciousness. Of course, our aforementioned four levels are an artificial and simplistic construct. One can refine them *ad nauseum*, but collectively they illustrate the depth and complexity of the social processes that mathematicians must engage in (and be trained in!) in order to respond to ethical issues in mathematics. Not every

[18] There is a Harvard-Stanford course being set up on AI and Ethics, with a budget of $23 million, and MIT is engaged in a billion dollar restructuring of its computing and AI studies around a core of ethical issues – amongst other things. Yes, that's US$1,000,000,000 they have raised, and are going to spend.

mathematician will engage in ethics at all levels,[19] but everyone should remain aware of their social responsibilities, acknowledge the existence of ethical issues in the mathematical context, and appreciate their complexities. Lawyers, medics, biologists, engineers, physicists, and computer scientists are aware of subject-specific ethics because they *know* they will encounter these questions as professionals. Comprehending the seemingly limitless uses of mathematics is difficult, and the ethical implications of modern mathematics sometimes depends on subtleties that only the mathematically trained can understand. We are the only ones who can understand the formulae, and *see beyond the formulae*.

However, the conclusion we feel is the most important for further study of ethics in mathematics is that we need the skills and knowledge of social scientists – sociologists, anthropologists, and psychologists – (*i*) to understand better the nature of mathematicians as individuals from the point of view of their particular psychological traits, and (*ii*) to understand the community values and behaviour, the 'tribal' behaviour of mathematicians. In particular we need to understand much better the particular social rules and behaviour patterns of mathematicians *with* mathematicians, and how mathematics do (and do not) communicate and interact with other groups of people, managers and non-mathematician work colleagues, both when working in teams and working in situations of conflict. Trying to broaden our understanding of how mathematicians can and should respond to ethical issues requires us to understand ourselves – and our communities – much, much better. There are social scientists who have spent decades refining tools to do exactly this, and we need them!

We feel that mathematicians have an array of social and mathematical values and patterns of behaviour as a group, and until we understand all of this much better we will not fully understand our own community or our own profession, and what makes it so different from our neighbours in computer science and physics. We do not understand well enough the motives, inhibitions, and driving mechanisms mathematicians have that need to be changed, or adapted in a way that will better enable them as individuals (and as a part of many different kinds of communities) to develop sensitivity and willingness to respond to ethical issues in their professional lives. Literally: learning – being better trained – to take on management roles and to engage with non-technical managers, to 'get a seat at the tables of power' is something we need to do … and something we need to be teaching our students. Only social scientists have the skills and training to analyse and explain to us how we work socially (if we can overcome *their* fear of mathematicians and mathematics, and *our* notoriously superior attitudes towards their disciplines). Only when we understand ourselves *in the new mathematical world of the 21st century* will we be able to prepare ourselves and our students to become sensitive to, and confront, both the ethical issues and the social responsibilities that come with our unique, ubiquitous, world changing powers.

We cannot leave these issues to professional ethicists and philosophers. For the sake of mathematics, and for the freedoms we enjoy, we must not just do nothing. No one else can do it, so we must.

[19] Not every mathematician will spend a lifetime solving differential equations, but we still teach it extensively and deeply to all mathematics students because they *need to know*, and because it is part of the common basis of the profession; we suggest that teaching ethics and the social skills required to *be* ethical mathematicians is much the same.

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About EiM

The Cambridge University Ethics in Mathematics Project does research into the ethical questions that arise when doing mathematics. We hope to teach mathematicians about the impact of mathematics in society, and to give them some of the tools and insight that they will need to prevent any harm from taking place.

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About EiM Discussion Papers

EiM Discussion Papers is part of the *Cambridge University Ethics in Mathematics Project*. The scope of these discussion papers is to stimulate informed discussion around ethical issues in mathematics, the teaching of ethics to mathematicians, and a corpus of case studies.

All discussion papers are subject to internal peer review.

To submit a discussion paper or to contact the coordinators, please send an email to ethics-dp@maths.cam.ac.uk.