ETHICS IN MATHEMATICS DISCUSSION PAPER 2/2018

University of Cambridge

Is there Ethics in Pure Mathematics?

Remarks about History and Sociology

After considering some historical aspects and combining our findings with recent results from economic sociology, we proceed to analyse the question of ethics in pure mathematics by considering various definitions for the subject independently.

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Abstract

After a look at some historical aspects of mathematics and a short detour to recent developments in economic sociology, we proceed to the question of ethics in pure mathematics. The German historian Wehler described constitutions as the bible of the democratic movements of modernity, a time when juridical declarations dominated international communication. This style of communication has changed since 1945 when international institutions increasingly began to communicate using numbers. The analysis of these societal shifts will naturally lead us to considerations on the quasi-theological standing of numbers, and to the role that various distinct definitions of pure mathematics play. Using Beckert's concept of imagined futures, we will outline how the relationship between pure mathematics and society could be studied.

By considering different definitions of pure mathematics, the question of ethics will be put into a broader context. We show that the issue of ethics in pure mathematics can be seen as a symbol of the tensions arising from secularisation, and through this, the need for (and the existence of) ethics within pure mathematics will be established.

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Introduction

Vague notions such as financial stability, objectivity and fairness are inherently hard to pin down mathematically. Despite this, many applied mathematicians and computer scientists show ambition to set these and other societal problems on a sound mathematical foundation. This is a natural response: mathematics is usually considered a tool for doing good and, as humans, we are inclined to help society using the tools of our trade. But when solving a problem, we typically have a choice between the quadrilateral of laws, norms, markets, and technology, including mathematics, to address it.

Today, exact and quantitative knowledge is assumed to be the prerequisite for successful and goal-driven actions. We like to measure success and to be target-oriented and do not want to be left in the dark. Knowledge gives us the power to manipulate the physical and social environment surrounding us, and so, using mathematics to describe societal problems has become the new reality. However, even though mathematics is the universal language for quantification, we increasingly hit practical barriers that are more complex than not having access to sufficient data, such as external interests and societal biases in the underlying data sets. The practical limitations are ubiquitous, often making navigating the quadrilateral indispensable when looking for a well-rounded solution.

All of this relates to applied mathematics, but what can we say about the purer areas of mathematics? Should a pure mathematician think about ethics?

Remarks about History and Sociology

In 1863 the first chair for pure mathematics was established at the University of Cambridge[34]. Today, more than 150 years later, philosophers study both applied mathematics [12,37] and the applicability of mathematics [2,46], but defining pure mathematics is still tricky, not only for professional philosophers but also mathematicians and students alike. When I asked students about their definition of pure mathematics, I received many answers, but there were four which I found particularly interesting:

- There is Hardy's definition. Pure mathematics is a pursuit for its beauty, detached from physical reality.
- There is the theologian who says that mathematics is studying the mind of God.
- The puzzler enjoys it because of the buzz of solving a perfectly-defined problem, like the artist enjoying the buzz of perfect beauty within the realm of mathematics.
- Moreover, there is the Hardy-esque response, "I do it because it is completely detached from reality."

All students provided different answers and thought their definition was the obvious choice. However, with no two definitions being the same, it is only obvious that it is not obvious. Before we continue, it could be useful to note that notable mathematicians of the past have considered variations of these positions. These definitions might appear odd at first, but they are not as rare as one might be led to believe.

When Galileo famously exclaimed that the book of the universe is written in mathematics[15], he was in good company. Later Descartes wrote in a letter to Marin Mersenne (15 April 1630) that "the mathematical truths which you call eternal have been laid down by God and depend on him entirely no less than the rest of his creatures" (quoted in Strickland [47], page 106). For a complete overview of historical figures, I refer to Bradley[8], but it should be clear that the idea of God somehow being the foundation of pure mathematics has a long historical tradition, and its impact can still be felt today. Sriraman[44] interviewed several researchers on the foundations of mathematics, depicting the deep metaphysical beliefs that (often in a (Neo-)Platonist form) are still with us today.

Others do not consider mathematics as the perfect image of nature, and instead see it as an approximation or entirely independent of the physical world altogether. From the early unpublished works of Gauss and Schweikart, over Bolyai's and Lobachevsky's treatises on hyperbolic geometry, to Riemann's famous and revolutionary 1854 lecture discussing manifolds, the Riemannian metric and curvature, our understanding of the physical world has changed rapidly. We have long understood that mathematics alone cannot decide if we live in a Euclidean or non-Euclidean world and that it is a task left to physics. Already Bolyai had a good understanding of the epistemological challenges that came with this new geometry when he wrote in a letter to his father that "[he] created a new and different world out of nothing". Unlike Kant had proclaimed earlier, human intuition ("Anschauung") was no longer the intuition for physical space.

This independence of physical realities is representative of a larger phenomenon. Thomason and Sorensen compared the developments in algebra following from Abel's and Galois' work on the insolvability of the quintic to the development of Romanticism, finding an "irony of romantic mathematics"[43]. Over time some mathematicians gave their profession a definition closer to the arts and logical puzzles. Putting it in the words of Andrew Wiles, is saying that "pure mathematicians just love to try unsolved problems—they love a challenge" [33], and earlier Hardy had already proclaimed that "the mathematician's pattern […] must be beautiful […] beauty is the first test, there is no permanent place in the world for ugly mathematics" (compare Hardy[19], page 14).

Mathematics and art have been in a long-lasting relationship. One only needs to look at Desargues' and Poncelet's work on projective geometry, Taylor's and Lambert's work on perspective drawing, and Brunelleschi's theory of perspective (for an overview see Hahn[17]). However, this newly found room for creativity was different because it was entirely independent of the physical world. It was more than studying God's mind, describing nature or merely abstract scholarly work. From now on mathematics could be art, and mathematicians to be seen as artists.

Having seen that the previous definitions of pure mathematics are not arbitrary but grounded in history, we can move to study our original question: What can we say about the relationship between ethics and pure mathematics? One way to tackle the problem of ethics would be from a purely professional context. Pure mathematicians are not only researchers, they are teachers, they are academics, they are colleagues, and hence they are bound to ethics like any other academic. Another way would be to attack the definitions themselves, which is, however, of little practical value. The results would always be subjective because we cannot tell which definition the correct one is. Hence, it would be better if we could tackle the problem of ethics without attacking the definitions themselves.

The study of mathematics teaches us to accept criticism of our work. Indeed, any good mathematician accepts it when there is a flaw in his argument. But the study of ethics is different, in some sense it is more political, more personal, maybe more subjective, and at the very least it appears to be more difficult to us as a human being. A student once said to me, “to study pure mathematics, is not to study politics. A pure mathematician need not be concerned with politics.” While applied mathematics is much more connected with politics, his words show something important: the first encounter with ethics should occur in a non-threatening context. Therefore, on a completely different level, it is not a bad idea to see what happens if we just accept the various definitions and use them as the origin of our study.

All definitions have in common that they give pure mathematics an independent right of existence, much as when Daston wrote that “we believe [...] pure mathematics is conceptually and for the most part historically prior to and independent of applied mathematics. Indeed, the very term applied mathematics tells all: in order to be applied, the mathematics must already exist, just as theory is ‘applied’ to practice” (compare Daston[13], page 221). If we accept Daston’s premise or a sufficiently similar one, such as pure mathematics exists without applied mathematics but is inspired by it, then what can we say about ethics in pure mathematics? Does a pure mathematician need to be concerned about ethics?

Within the philosophy of mathematics, one can identify three main paradigms[42]. There is

- the ontological paradigm, which aims for vertical separation of our knowledge (compare Plato),
- the epistemological paradigm, which seeks to understand its relationship to our consciousness,
- and the anthropological paradigm, which views knowledge in its social and anthropological context.

It is useful to observe that two of the definitions of pure mathematics naturally fit into these paradigms. Pure mathematics as an exploration of God’s mind fits into the ontological paradigm, while Hardy’s definition fits better into the epistemological paradigm. It can be argued that the third definition fits into the anthropological paradigm if one considers the inspirations for an adventure in art. Each of the paradigms and definitions brings its challenge in the quest for ethics in pure mathematics — challenges that we will have to analyse if we want to find an answer. In doing so, we will have to take some further excursions. Hence, let us take another stroll through the history of mathematics. On our stroll, we will not only see why it is not sufficient when social scientists and philosophers study the mathematical community but why it is necessary for us mathematicians to take part in the discussion. We can learn something about our subject.

When Gray[16] asked if “the increasing abstraction of mathematics led to a sense of anxiety” for 19th-century mathematicians, he was more concerned with analysing the mathematical community as with the larger societal implications. In his view, mathematics was defined by a “growing appreciation of error,” which led mathematicians to seek new foundations for their subject. Thus, he argues, the mathematical sciences of the 19th century should not only be seen as innovative but also in view of the anxiety this venture entailed. I believe that the foundational crisis of the long 19th century, together with the subsequent construction of an ivory tower of mathematical abstraction, allowed Hersh to conclude that “in pure mathematics, when restricted just to research and not considering the rest of our professional life, the ethical component is very small"([21], page 22). One could go even further. Shaposhnikov[40,41] showed that the study of pure mathematics could fulfil the functions of theology in a time of secularised science. He argues that "theology was present in modern mathematics not through its objects or methods, but mainly through popular philosophy, which absolutised mathematics. [...] Modern pure mathematics was treated as a sort of quasi-theology [and] a long-standing alliance between theology and mathematics made it habitual to view mathematics as a divine knowledge, so when theology was discarded, mathematics naturally took its place at the top of the system of knowledge" ([40], page 1).

It seems to me that throughout the centuries pure and applied mathematics seemingly separated on a psychological level and because of the two world wars the separation rapidly deepened for some mathematicians. A representative is Hardy and his 1940 Essay "A Mathematician's Apology." Hardy pointed out that mathematicians usually do not "glory in the uselessness of their work," but that "mathematicians may be justified in rejoicing that there is one science at any rate and that their own, whose very remoteness from ordinary human activities should keep it gentle and clean." For Hardy pure mathematics was disconnected from the physical world. Indeed, he said of himself to "have never done anything 'useful'. No discovery […] has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world" (see Hardy[19], page 49).

In contrast to his definition are the many events taking place in the public sphere. After 1945 the world experienced a rise in international organisations communicating using numbers. While juridical declarations predominantly defined previous communication, numbers started to creep into the picture[11]. Hence, when some pure mathematicians turned to one side, the world turned to the other. For a while, society has not made a distinction between pure and applied mathematics, and it does not use mathematics as if this distinction exists. Much of society uses numbers in a similar quasi-theological fashion as some mathematicians use pure mathematics. There is, however, a substantial difference in how mathematicians and the public perceive it. Numbers in the public sphere always have a normative component[11]. They are used to rank, used to compare, used to show improvement and decline. The 2016 Paris agreement states that we aim to keep the global rise in temperature below 2 degrees Celsius[49], the European Union debated a ceiling on the number of refugees[53], and poverty is of course classified by how much you earn[51]. On an international level, you are poor if you make less than $1.90 a day. Such numbers are normative, and they define international communication as the new global language. A language that everyone understands, seemingly free of traditions and value judgments. All these numbers contain an evaluative component. The higher your country's GDP, the better. Numbers and measures used in real-world scenarios are not only numbers, they are an expression of our understanding of the world, of future goals and our past.

All of this is entirely different from why mathematicians use numbers. For many pure mathematicians, their research does not entail a normative component even though it changes every time their research is used in the real world. This suggests an increasing gap between how mathematicians think about their field and how society does, or at least how social scientists do, a gap which is deepening over time due to the communicative power of numbers. Using numbers is easy. They go into "a natural symbiosis with both narrative and pictorial communication" from which they deduce their "reality defining power", but at the same time numbers anonymise said power, and there seems to be no overarching instance bearing the responsibility[11]. All responsibility is lost in the pipeline of communication, lost somewhere between the scientist creating the number and the newspaper article. Consequently, by referring to numbers, we, as a society, not only give up some responsibility but also control.

By now the reader might have noticed that we talk about numbers in the public sphere and not statistics, and there is an excellent reason for it. While for mathematicians it is clear that most such numbers are statistics or other results from mathematical models, this is not so clear for the mathematical layman. A confusion that is fundamentally connected with technological progress. The Archbishop of Canterbury, Justin Welby, said, "there are politicians driven by short-term pressure from the change in information technology, which gives you information without relationship and gives you no time to respond"[54]. Our modern communication is intermingled with technological advances, and, hence, it is intermingled with mathematical research. Modern technology makes it easy to put out information without context and often numbers are part of this contextless information. While we cannot make without numbers, we still must consider where it leaves us as (pure) mathematicians. After all, for the public we are the ones studying the numbers, we are the ones providing the basis for future technologies. The many methodological difficulties involved when quantifying real-world phenomena (for an overview see Mayntz[29]) are not easily understood, and one could argue that even when mathematicians understand the limitations of impartiality of their mathematics, it does not imply that the public understands it. Often it looks as if applied mathematics is treated the same as pure mathematics. If this is the case, we should ask how this could have happened. A question that is intimately connected to another question: Why do we act when we cannot know the future?

For a long time, social scientists have known that past experiences influence the present, but a recent phenomenon in the social sciences is to put more focus on the future. We all know that nobody invests money if he does not expect to make money out of his investment, but how does the underlying decision process look like? Beckert[3] argues that imagined futures and fictional expectations are critical to decision making in a capitalist environment. Our imagination of the future creates a willingness and readiness to act, and therefore fictional expectations, i.e. what we imagine the future to look like, create the present. All imagining happens in a socially constructed environment, and part of the social construct is the use of mathematics. Mathematical models are examples of instruments that help us to imagine the future. There is substantial political and economic power embodied in our fictional expectations, and consequently in the methods and instruments used to create them. Among these instruments, mathematics is deemed the most credible and influential. We as mathematicians push such stories even when we do not notice it. After all, mathematics is the queen of the sciences, isn't it? It is different from the other natural sciences; we can prove theorems.

Beckert does not argue that economic actors are irrational, but instead, an actor behaves rationally based on an imagined future. Furthermore, he explains that, even though our imagined futures are fictional, they are in some sense real: their impact can be felt through our actions every day. The origins of imagined futures are diverse: institutions, social status, social networks, calculative instruments, cultural structures, the interdependence of expectations, and protention are among them[4].

The Origins of Imagined Futures of Mathematics

In what follows, we provide a short overview of how the origins of imagined futures of mathematics could be analysed.

Institutions

We often have an overly romantic image in our mind when thinking about the great mathematicians of the past. It may be Galileo looking through his telescope, or Abel who died far too young at the age of 26 and went largely unrecognised during his lifetime. Hermite famously said, “Abel has left mathematicians enough to keep them busy for five hundred years” (quoted in Bell[5], page 10). The reality, however, was often different. Galileo worked at the University of Pisa since 1588 and later at the University of Padua, where he had to tutor students and teach classes in return for his stipend. Like many scientists, Galileo sought funding and patronage from wealthy families, in his case the Medici, and he was a member of the Accademia dei Lincei which published his works and provided a regular space for discussion for a small group of scientists including Johann Faber, Luca Valerio and Giambattista della Porta[52].

It is undeniable that institutions have played an essential part in the development of science and mathematics. However, with institutions comes politics, not only departmental politics but often national and international politics. One prime example is the development of scientific institutions in France. In the time of the Old Regime, science and European colonial and imperial expansion were deeply connected[30], and even today the variety of research and funding institutions, and the professional societies, are not shielded from politics. Indeed, they play an integral part. The roles they play, and often give themselves, differ throughout history. One striking example is Heffters lecture at the German Mathematical Association during the Great War.

“It almost requires an excuse [...] having to do purely scientific work in the face of the incomparable war and even venturing their results from the closed study room into the public eye. And yet, especially in this war, science, especially the technical and natural sciences, and thus indirectly also mathematics, has been able to celebrate unprecedented triumphs. But no science would be able to solve tasks that are suddenly posed from the outside if it had not at any time, kept in mind the development that seemed good for internal reasons, and proceeded unperturbed.” (compare Heffter[20], page 1, my translation)

Even when mathematicians wish to be unconcerned and unperturbed of politics, they are not, as Heffters introductory words make clear. Institutions and politics play an important role in the development of science, and they shape the imagined futures of mathematics. Heffter himself touches on an imagined future when he says that mathematics should proceed unperturbed.

Social Position and Social Networks

Already Bourdieu argued that our social position forms a “habitus” through which we guide our expectations[7]. The effect of social networks on decisions and expectations has been analysed in the context of financial markets[45], but it has yet to be examined with respect to the mathematical community. The mathematical community around the world is, in some respect, quite diverse, and while viewed from another angle surprisingly homogeneous. It would not be surprising if such a study entailed some surprises.

Calculative Instruments and Interdependence of Expectations

“[All] calculative instruments unite that they create expectations regarding future developments, including expectations for the actions of third parties. They allow [us] to tell a believable story to someone trusting the instrument” (see [4], page 6, my translation). Mathematics lies at the heart of calculative instruments. It may be the probability theory in economic forecasts or nowadays machine learning. We only need to look at the Social Credit System proposed by the Chinese government[39], which makes heavy use of machine learning and big data techniques[32].

While mathematics often helps to shape imagined futures of the public, governments, institutions or an individual, it is also their expectations of mathematics which influence our, i.e. the mathematicians', imagined futures and expectations. As Beckert puts it, "Expectations are generated from the observation of others' expectations and are therefore genuinely socially constituted (see Beckert[4], page 8). This phenomenon is called the interdependence of expectations, and it shows us that, when applying mathematics to a societal problem, it is not always crucial whether our mathematical model is correct but how others perceive it.

Cultural Structures

McCloskey analysed the importance of Bourgeois values and the corresponding arising expectations in capitalism[31], and others have studied the concept of virtue in the age of enlightenment[26]. One approach would be to investigate similar cultural structures within the mathematical community. Has a mathematician's concept of virtue changed over the last 300, or maybe 2000 years? In what other values originates a mathematician's imagined future? We have already briefly touched the romantic image of mathematics, and the definitions that lie at the basis of our ethical analysis suggest this to be a fruitful approach.

Protention and Historicity

Edmund Husserl argued that human perception is made of three temporal aspects, retention, present and protention[24]. Retention describes a temporally extended past: humans use past events to understand the present and future. Protention, on the other hand, describes the fact that our anticipation of the future is dependent on patterns of the past.

We have already seen that the way we understand and do mathematics is not disconnected from the past. Mathematics is a field of long historical and cultural tradition. Are compass and ruler constructions still used to answer Parmenides scepticist agenda? Probably not, but the quasi-theological underpinnings of mathematics suggest broader historicity in our subject's imagined futures. There are studies on the interplay between mathematics and sociological ideas (see Tasic[48] for an example of the interplay with postmodernism). However, the historicity within mathematics has yet to be analysed with respect to its interplay with other societal developments in the context of ethics.

Ethics in Definitions

Having now seen how mathematics can shape society through expectations and imagination, we can turn back to our original question: Should a pure mathematician think about ethics? Is there (room for) ethics in pure mathematics? Obviously, if one accepts these premises, then there is ethics in pure mathematics. But what if one does not recognise these premises?

When we use Hardy's definition of pure mathematics, and ignore the effect of imagined futures, we can certainly tell stories like (pure) mathematics is at worst neutral but never bad. However, such a definition must allow for a fluid content of pure mathematics. We, as pure mathematicians, often cannot tell when or if our research will have practical implications. Hardy could not tell it either. Indeed he was wrong about some of his research[38]. Parts of what we consider pure mathematics may only be temporarily independent of the physical world. And so, a mathematician using this definition might face ethics at one point in his life when his pure mathematics suddenly becomes applied mathematics.

Therefore, we could fall back to an alternative definition. One such alternative is given by Brown[9] when he argues the difference between pure and applied mathematics is a question of focus. A mathematician should ask only one question: What is the focus of the problem he is studying? If he is interested in gaining mathematical insight, then it is pure. If, on the other hand, the goal is to gain a better understanding of a worldly, not necessarily physical, phenomenon, then it is applied.

Defining pure mathematics in this way implicitly accepts a possible real-world significance, and thus makes room for ethical considerations. However, Hardy's and Brown's definitions are not the only alternatives. As we have seen, there are other ways to define or think about pure mathematics, and some are intimately connected to theology. But before considering what happens if we define pure mathematics as the study of God's mind, we must look at the relationship between science and theology in general. We must clean up our language and better understand our assumptions.

Alexander[1] compares four major models for describing the relationship between science and theology. Before we continue, let us make a short comparison and study what happens when we substitute pure mathematics for science. It is a seemingly impossible task because all results will depend on our definition of pure mathematics, but let us continue in the hope to gain some insight.

The conflict model describes science and theology as being in natural conflict. We first observe that mathematics and religion are not in inherent conflict. Looking at ancient and medieval history shows that for a long-time mathematics and theology were indeed deeply connected, and even nowadays mathematics is, for some people, the exploration of God's mind. So, the conflict model seems not suitable for pure mathematics.

The NOMA model ("Non-Overlapping Magisteria") states that science and theology study different domains and have therefore no connection. Whether this is an appropriate model to describe the relationship between pure mathematics and religion, depends entirely on our definition of pure mathematics. If we define pure mathematics as wholly disconnected from the real world, then this might be true because in this case, we are not studying God's creation. On the other hand, if we, for example, use Brown's definition, then pure mathematics seems to explore ideas from a different angle, and so they are not entirely disconnected.

Fusion models "tend to blur the distinction between scientific and religious types of knowledge altogether or attempt to utilise science in order to construct religious systems of thought, or vice versa" (see Alexander[1], page 3). This type of thinking is not as well supported in modern mathematics as it was in ancient and medieval times. Nowadays, pure mathematics seeks foundations within itself and aims not to influence religion. Even when it is used as a quasi-theological substitute, it is no longer as heavily influenced by theological ideas. More often it appears as a simple search for beauty and truth.

Finally, the complementary model argues that science and religion study the same phenomena from different angles. Both subjects can consequently give rise to different explanations and answers, which can be complementary to another. As it is in the case of science, this seems to be the most supported for mathematics, but of course, it does not capture every detail. It is indeed possible to believe in religion, and work as a mathematician, e.g. consider Sriraman's conclusion after interviewing several mathematicians:

"Like Leibniz, these mathematicians argued that there was no dichotomy between faith and reason. Two of these mathematicians mentioned that it was possible to be Christian and a mathematician if one did not interpret the Bible literally. One of the mathematicians stated that duality ceased to exist when he created mathematics and reported experiencing oneness with the object of creation. This mathematician mentioned that a genius like Ramanujan must have experienced a oneness with the universe of positive integers, which perhaps led to the deep insights that he had about numbers. The discussion of compatibility between science and religion naturally led the mathematicians to compare their work to that of an artist“ (see Sriraman[44], page 144).

For the remainder of this essay, we will accept the complementary model and assume that a mathematician, who is exploring God's mind through mathematics, is not doing it to justify a set of religious beliefs. Which is not to say that it is the only way of proceeding with this analysis, but we must take a stance at this point. The lines between pure and applied mathematics are not fixed, and history has shown that neither is the relationship between religion and mathematics[25]. Just like one model works well when we accept certain premises, one derivation for ethics works well if we accept other assumptions. It is a task for future research to analyse what happens under a different set of assumptions.

If pure mathematics is defined as the study of God's mind, then one possible route could be to exclaim that aesthetics and ethics are one. Where you find one, you will find the other. This, however, requires a Wittgenstein-like definition, something that might not be of practical use in the daily life of a pure mathematician. What else can we say without pondering too much about our definition of God's mind and our definition of ethics?

Regarding the first two definitions, we have seen that it is possible to argue along the lines of social influence, an argument which can be unsatisfactory from an aesthetical point of view. On the other hand, using purely philosophical arguments presented itself with difficulties because of the vague nature and absolute status of pure mathematics. Maybe the necessity to consider ethics in pure mathematics lies in the quest for knowledge itself, no matter whether it is the study of God's mind?

We now define the study of pure mathematics as an artistic quest for knowledge. What can we say in light of this definition?

Arguably, famous artists have an ethical obligation to society, but mathematicians rarely reach the same level of fame. Furthermore, an artist's ethical position is easier to understand than a mathematician's. If a mathematician wishes to express an ethical position through his research, it is more difficult for him to reach the same exposure. Adapting Beckert's work suggests that his exposure will depend heavily on his and others imagined futures, dependent on the envisaged use of his mathematics, something which can be easily hidden. The artist mathematician might not think in terms of use, and even when he tries to imagine future uses, he can never be sure. The way to understand ethics in the artist's or theologian's definition of pure mathematics is not through sociological ideas, nor historical analysis. We must revert to literature if we want to understand its relation to ethics and morality.

“Imaginative construction, verbal or visual, works to make present an aesthetic object that allows itself to be contemplated from a perspective or perspectives other than those of the artist's own subjectivity. Art makes possible a variety of seeings or readings; it presents something that invites a time of reception or perception, with the consciousness that there is always another possible seeing/reading. [...] Therefore [it is] never reducible to an instrumental account [...] Instead, there is an indefinite time opened up for reception and interpretation: the object is located outside the closures of specific conflicts and settlements of interest” (see Williams[50], page 13).

One of the essentials of art is that it can be seen from different perspectives. It invites the struggle between the secular (a world in which everything can be explained from a human perspective) and the non-secular (a world in which some things cannot be explained from a human perspective)[50]. It invites the unexplainable and the imaginative, and hence the artist's definition of pure mathematics is remarkably like the theologian's definition. Here, too, lies an invitation to the unexplainable and imaginative.

For the Romanticist artist, there is no art criticism outside of art, and "poetry can only be criticised by way of poetry. A critical judgment of an artistic production has no civil rights in the realm of art if it isn't itself a work of art" as Schlegel wrote in his Critical fragment 117. Art criticism and reflection happen within art, and it is never an explicit criticism. "For the value of a work depends solely on whether it makes its immanent critique possible or not […] and […] if a work can be criticised, then it is a work of art; otherwise it is not - and although a mean between these two cases is unthinkable, no criterion of the difference among true works of art may be contrived either" (see Benjamin[6], page 160). Isn't this astonishingly similar to mathematics? Often, we think that our mathematics can and should only be criticised within the realm of mathematics itself. We critique (and judge) proofs by its originality, often only comprehensible within the language of mathematics, and if we find our critique beautiful, we often deem the proof itself as beautiful if not valuable. To put it in Schlegel's words, "True criticism can [...] take no notice of works that contribute nothing to the development of art" (Schlegel, Jugendschriften, II: 423, quoted in Benjamin[6], page 160).

The Romanticist spirit is that of becoming, the value of ideas and art is not time-transcending. Instead, it is fixed in time and place. Artistic works are nothing but historical fragments, symbols, and, in some peculiar way, historical sources — a situation that is very similar to that of mathematics. Once a part of mathematics is matured, we treat it as an authentic fragment and use it in the creation of new mathematics. A theorem is a historical source helping us to create something new. It is inherently historical, that is why we give names to important theorems. We put it in time and place, honour its creator, and try to establish timelessness, while ultimately failing because the later beauty of the theorem will always be different from the moment of its creation and discovery.

Here, one finds ethics at the very heart. By accepting pure mathematics as a form of art (or as the study of God's mind), we allow an inherent instability but banish secular dogmatism. Gauss might have decided not to publish his differential geometry because he feared the outcry in a Euclidean, Kantian, world, but nowadays, mathematics has a welcoming attitude towards the non-secular, the rationally unexplainable beauty of ideas. It is, hence, sweepingly different from the developments in the secularising western world, a world in which much is valued in functionalist terms, and which loses the vocabulary for the unexplainable, the unthinkable (compare Williams[50]). For many mathematicians the world has never been disenchanted.

Pure mathematics is often assumed to be distanced from society, but a closer analysis shows that it influences the public's imagination, understanding of all mathematics and, hence, all numbers in the public sphere. A discipline, which is both distanced from society but close enough to affect it, is inherently ethical.

Does Thinking about Ethics Enhance Mathematics?

One could ask if thinking about ethics enhances our understanding of mathematics. Does it help us in our venture to find better proofs? Does it help us to create or discover more beautiful mathematics?

Thinking about ethics led us to examine the history of our subject and to appreciate the humanity in it. For some, this might create an additional component of beauty. In the end, it could help us to demystify mathematics for the public. Ethics in pure mathematics is a problem without a straightforward objective answer. It is deeply connected to the plurality of reasons why we study pure mathematics and how we see its relationship with other disciplines. It is a suitable ground for reflection among us mathematicians, a place to think about mathematics on the meta-level.

The Roman Catholic Theologian Bernard Lonergan suggested studying other disciplines in the light of mathematics.

"One has also to examine mathematics, and discover what is happening when one is learning it and, again, what was happening as it was being developed. From reflecting on mathematics one has to go on to reflect on natural science, discern its procedures [...] From the precision of mathematical understanding and thought and from the ongoing, cumulative advance of natural science, one has to turn to the procedures of common sense, grasp how it differs from mathematics and natural science, discern its proper procedures, the range of its relevance, the permanent risk it runs of merging with nonsense" (see Lonergan[27], page 260).

Our study has been in light of his idea. From mathematics, we went on a stroll through philosophy, sociology, and theology, only to finally come back to mathematics itself.

We can argue that in an age of increased mathematical specialization, when most papers are read and understood only by a fraction of all mathematicians, the aesthetic argument (pure mathematics as an art form, worth because of its beauty) and the future-value argument (pure mathematics is worth because of its later practical implications), are hard to defend. All of this provides challenges to pure mathematics, and hence in the future, our definition of pure mathematics might be different. The question "Is there ethics in pure mathematics?" could be answered differently by a future generation. While in this discussion paper we could not provide the reader with an algorithm for ethics, we succeeded in showing that it is fruitful to think and reflect on ethics. We proved that there is a room for considerations relating to ethics and pure mathematics.

There is plenty opportunity for future research, especially regarding the connection with applied mathematics and the origins of imagined futures and their relationship to mathematics. As we have already pointed out, interesting case studies include the institutionalisation of mathematics and its relationship to ethical questions. We already know that scientific institutions are deeply connected with politics and that research is influenced by it (for an overview and historical example, see Martin[28] and Hahn[18] respectively).

An alternative direction would be to explore the idea of virtue. Has a mathematician's understanding of virtue changed? The concept of virtue is closely linked to ethics. Virtue has been studied in the context of politics and enlightenment[26], but it has yet to be studied in the context of mathematics. The question, "Is there ethics in pure mathematics?", suggests itself as a suitable theme for this research.

Irrefutably, society and religion have influenced the development of mathematics, and the development of mathematics has affected society and our thinking. It should be reason enough to make a mathematician worry about ethics. Even when our research programme might not have an immediate impact on society, the way we think about it will have a long-term impact. After all, the way we think about pure mathematics affects how society and our students perceive it, which even happens when research in pure mathematics is seemingly disconnected from everything. How we, as researchers, define and perceive pure mathematics could have unforeseen consequences in the long-run.

Before we finish, let us look at a little proof. Vern Poythress[35] first proposed its structure, and then Howell adapted it to the question of ethics in mathematics[23]. Consider the statement "C: ethical considerations should have no bearing on the practice of mathematics." If we accept C as an axiom, it follows that mathematical practice ought not to be influenced by ethical claims, and so it must not be influenced by C itself. Thus, ethical considerations should influence the practice of mathematics. Do you agree?

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